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A Theoretical Variation of the Wind Profile Power-Law Exponent as a Function of Surface Roughness and Stability

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The variation of the wind profile power-law exponent with respect to changes in surface roughness and atmospheric stability is depicted using the formulation of Nickerson and Smiley for specifying the vertical variations of the horizontal wind. The theoretical estimates of the power-law exponent compare favorably with power-law exponent data from various sources.

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TECHNICAL NOTE

A THEORETICAL VARIATION OF THE WIND PROFILE POWER-LAW EXPONENT AS A FUNCTION OF SURFACE ROUGHNESS AND STABILITY

(*First received* 2 *February* 1978 *and in final form* 27 *July* 1978)

Abstract – The variation of the wind profile power-law exponent with respect to changes in surface roughness and atmospheric stability is depicted using the formulation of Nickerson and Smiley for specifying the vertical variations of the horizontal wind. The theoretical estimates of the power-law exponent compare favorably with power-law exponent data from various sources.

For numerical and mathematical convenience, the wind profile power-law is often used to estimate the horizontal wind speed, U_2, at a height above the ground, Z_2, given the horizontal wind speed, U_1, at a reference height, Z_1, such that:

$$U_2 = U_1(Z_2/Z_1)^p \tag{1}$$

where p, the power-law exponent, is usually specified as a function of stability. Ready access to small computers permits direct computations of wind speed as a function of height using the semiempirical formulations for the nondimensional wind shear presented by Nickersen and Smiley (1975) and Benoit (1977). However, power-law profiles still find wide acceptance in simple Gaussian plume dispersion models. Touma (1977) recently commented on the risk of using exponent values which are stated only as a function of stability since the reported values vary depending on the site location. The purpose of the following discussion is to demonstrate that the variation in the reported exponent values can be explained as primarily due to variations in surface roughness.

Panofsky *et al.* (1960) investigated the stability and the surface roughness dependence of the profile exponent values by using a semiempirical formulation for the nondimensional wind shear,

$$\phi_m = (kZ/u_*)(\partial U/\partial Z),$$

by estimating the exponent values as:

$$p = (u_*/Uk)\phi_m \tag{2}$$

where k and u_* are the von Karman constant and the surface friction velocity. The results of Nickerson and Smiley (1975), Benoit (1977) and Businger *et al.* (1971) can be used to estimate (u_*/Uk) and ϕ_m for use in Equation (2) as:

For unstable conditions,

$$(u_*/Uk)^{-1} = \left\{\ln(Z/Z_0) + \ln\left[\frac{(\zeta_0^2+1)(\zeta_0+1)^2}{(\zeta^2+1)(\zeta+1)^2}\right] + 2[\tan^{-1}(\zeta) - \tan^{-1}(\zeta_0)]\right\}$$

$$\phi_m = (1-\gamma Z/L)^{-1/4}$$

where

$\gamma = 15.0$

$\zeta = (1-\gamma Z/L)^{1/4}$

$\zeta_0 = (1-\gamma Z_0/L)^{1/4}$

L = the Monin–Obukhov scaling length,

and for stable conditions,

$$(u_*/Uk)^{-1} = \ln(Z/Z_0) + \beta Z/L$$
$$\phi_m = 1 + \beta Z/L$$

where $\beta = 4.7$.

Based on the results of Weber *et al.* (1975), Equation (2), with the above substitutions, should perform well for heights up to 160 m in near-neutral conditions. These formulations assume that conditions are steady-state with unobstructed flow. Also, they are restricted to that portion of the atmospheric surface layer where $\partial U/\partial Z$ is greater than zero. For extremely unstable or stable conditions, p estimates at heights of 100 m may not perform well, since the conditions are not likely to be steady-state. The substitutions into Equation (2) reveal that the exponent, p, is a function not only of stability and surface roughness but also height. Average exponent values over a layer from Z_1 to Z_2, $\bar{p}$, are best calculated numerically since the variation of p with height is not straightforward. From an analysis for surface roughness ranging from 0.01 to 1.0 m and heights up to 100 m, it was found that $\bar{p}$ could be adequately approximated by using one height, Z'. Using the average height of the layer for Z' turned out to be too high and using the geometric mean height of the layer for Z' turned out to be too low; whereas, using the average of the two for Z' seemed to work well.

Figure 1 compares the theoretical predictions of the power-law exponent with power-law exponent data from three sources. The data for the 1968 Kansas Field Program, Izumi (1971), where z_0 was approximately 10 cm, reported hourly values for the Monin–Obukhov length scale. The values for p were derived from the reported wind speeds for the two levels, 32 m and 2 m, using:

$$p = \frac{\ln(U_2/U_1)}{\ln(Z_2/Z_1)}. \tag{3}$$

Touma (1977) reported average values of the profile exponent, p, for each Pasquill stability class. The exponent values were calculated using the 60 m and 10 m meteorological tower data collected at several nuclear power plant sites

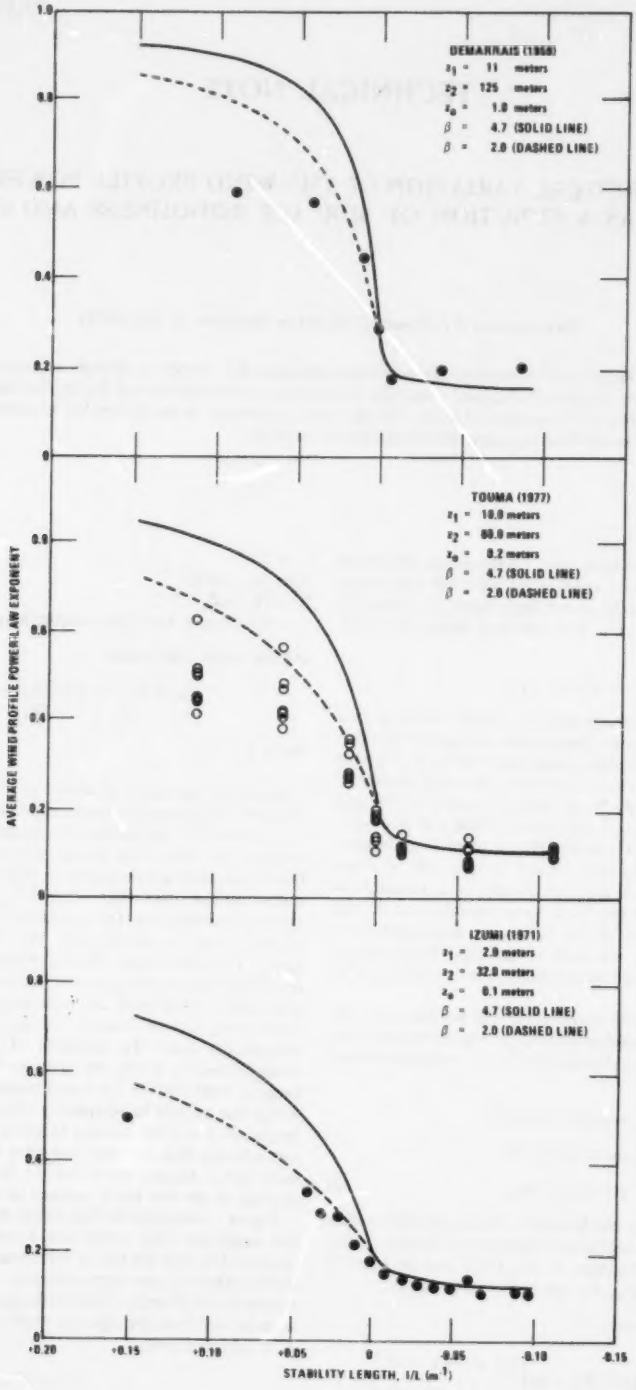

Fig. 1. Comparisons of the theoretical predictions of the power-law exponent with power-law exponent values from three field measurement studies. Exponent values are average values for layer from z_1 to z_2.

located in rolling terrain. The data used in Fig. 1 is from Table II of Touma (1977) from which the surface roughness was estimated to be approximately 20 cm. The Pasquill stability classes were assigned by Touma (1977) based on the vertical temperature gradient criteria given in Regulatory Guide 1.23 (1972). Values of the stability length, L, were estimated using the relationships given by Liu *et al.* (1976). These values are best fit curves to Fig. 4 of Golder (1972) which relate surface roughness and Pasquill stability category to the Monin–Obukhov stability length.

DeMarrais (1959) reported average values for the profile exponent, p, as a function of temperature lapse rate from the 125 m and 11 m levels of the Brookhaven National Laboratory meteorological tower. The surface roughness was estimated to be approximately 1 m. Using the lapse rate criteria given in Regulatory Guide 1.23, average exponent values were calculated for each Pasquill stability class. These exponent values were then related to stability length, L, using the relationships by Liu *et al.* (1976).

In Fig. 1 the comparison plots indicate that during stable conditions the theoretical predictions agree with the data if β is set to 2.0 rather than 4.7. Sethuraman and Brown (1976) suggested that β approached 1.6 for very stable conditions. The predictions of the profile exponent are slightly high for the Kansas data during unstable conditions. The scatter in the data comparisons using the Touma (1977) and DeMarrais (1959) data may be related to the difficulty in estimating stability lengths for Pasquill stability classes based on temperature lapse rates. In general, the data comparisons indicate that estimates of the wind-profile exponent can be made with β set to 2.0.

Figure 2 shows the comparison of the theoretical predictions with variations in surface roughness using the exponent variation suggested by Counihan (1975) for adiabatic conditions (near Pasquill stability class D). The relationships reported by Liu *et al.* (1976) were used in developing Fig. 2 in order to estimate the stability length for each Pasquill stability class as a function of surface roughness. The theoretical predictions for stability class D seem to agree very well with Counihan's results. Table 1 list the theoretical power-law exponent values extracted from Fig. 2 for several surface roughness values. The L^{-1} values as suggested by Liu *et al.* (1976) are also given in Table 1.

In conclusion, the results depicted in Figs. 1 and 2 indicate that during stable conditions the power-law exponent is mostly a function of stability and only a weak function of surface roughness. During unstable conditions, however, the power-law exponent is mostly a function of surface roughness and only a weak function of stability. The theoretical estimates of the power-law exponent were within 30% of the data presented. The suggested dependence of the power-law exponent on surface roughness, as well as stability, seems to explain the differences in the reported exponent values derived from Field data.

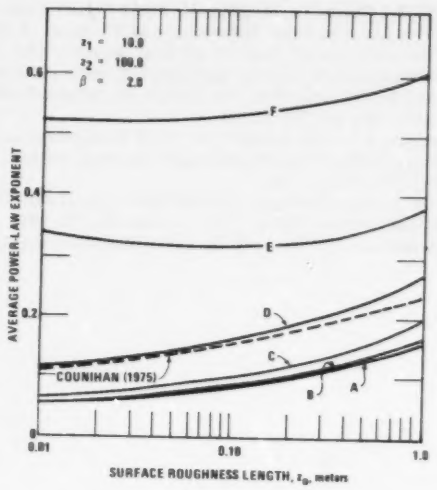

Fig. 2. Variation of the power-law exponent, averaged over layer from 10 to 100 m, as a function of surface roughness and Pasquill stability class. Dashed curve is result suggested by Counihan (1975) for adiabatic conditions which should agree with stability class D.

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Table 1. Variations of the power-law exponent, $\bar{p}$, as a function of Pasquill stability class and surface roughness, z_0

Stability	$z_0 = 0.01$ m		$z_0 = 0.10$ m		$z_0 = 1.00$ m		$z_0 = 3.00$ m	
	L^{-1} (m^{-1})	$\bar{p}$	L^{-1} (m^{-1})	$\bar{p}$	L^{-1} (m^{-1})	$\bar{p}$	L^{-1} (m^{-1})	$\bar{p}$
A	−0.1405	0.05	−0.1109	0.08	−0.0875	0.17	−0.0781	0.27
B	−0.0848	0.06	−0.0571	0.09	−0.0385	0.17	−0.0319	0.28
C	−0.0329	0.06	−0.0163	0.11	−0.0081	0.20	−0.0058	0.31
D	0.0000	0.12	0.0000	0.16	0.0000	0.27	0.0000	0.37
E	0.0329	0.34	0.0163	0.32	0.0081	0.38	0.0058	0.47
F	0.0848	0.53	0.0571	0.54	0.0385	0.61	0.0319	0.69

Z_1 and Z_2 are 10 m and 100 m with β equal to 2.0. The L^{-1} values as suggested by Liu *et al.* (1976) are given for reference.

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